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PROBLEM OF THE WEEK

11/6/01 due NOON 11/20/01

CAN YOU GIVE US A SOLUTION?

Problem No. 11 (Fall 2001 Series)

Let $\{a_0, a_1, a_2, \dots\}$ be a non-zero sequence having period N , that is, $a_{k+N} = a_k$ for all $k = 0, 1, 2, \dots$. Show that

(1) $\sum_{k=0}^{\infty} a_k z^k$ is a rational function for $|z| < 1$,

(2) $\sum_{k=0}^{\infty} a_k$ diverges, but

(3) $\lim_{z \rightarrow 1^-} \sum_{k=0}^{\infty} a_k z^k$ exists if and only if $\sum_{k=0}^{N-1} a_k = 0$; find the limit.

A panel in the Mathematics Department publishes a challenging problem once a week and invites college & pre-college students, faculty, and staff to submit solutions. The objective of this is to stimulate and cultivate interest in good mathematics, especially among younger students. Solutions are due within two weeks from the date of publication. They can be faxed to (765) 494-0548 or sent by campus or U.S. mail to:

PROBLEM OF THE WEEK, **8th Floor**, Math Sciences Bldg., Purdue Univ.,
West Lafayette, IN 47907

Solvers should include their name, address, and **status at the University or school**.

The names of those who submitted correct solutions will be posted in the Math. Library, along with the best solution. Every Purdue student who submits three or more correct solutions will receive a Certificate of Merit. A prize fund of \$150.00 will be distributed among the Purdue undergraduates who have contributed at least six correct solutions for the total fall 2001 series.