PROBLEM OF THE WEEK

Solution of Problem No. 2 (Fall 2001 Series)

Problem: Given a line ℓ and points P, Q in a plane with ℓ on opposite sides of ℓ.

a) Determine a point R on ℓ which maximizes $\big||PR| - |QR|\big|$.

b) Does such a point R always exist?

Solution (by Damir D. Dzhafarov, Fr. MA)

Reflect P over ℓ, denoting its image on the other side P', and let R be any point on ℓ. From the triangle inequality it follows that $\big||PR|-|QR|\big| = \big||P'R|-|QR|\big| \le |P'Q|$ so that the left side of the inequality is maximized when it equals the right. This occurs when the three points P', Q, and R are collinear. Thus, constructing R by producing $\overline{P'Q}$ until it crosses ℓ maximizes $\big||PR - |QR|\big|$. However, if P and Q are equidistant from ℓ then $\overline{P'Q}$ will be parallel to ℓ and it will not be possible to find such an R by the above method. In this eventuality $\big||PR| - |QR|\big| < |P'Q|$ with the left-hand difference getting arbitrarily close to $|P'Q|$ for distant enough R. Hence, no R makes $\big||PR| - |QR|\big|$ a maximum in this case.

Also solved by:

Undergraduates: Eric Tkaczyk (Jr. EE/MA)

Graduates: Tamer Cakici (ECE), Ashish Rao (EE), K. H. Sarma (Nuc E), D. Subramanian (CHME)

Faculty: Steven Landy (Phys. at IUPUI),

Others: Michael Hamburg (Sr. St. Joseph's H.S., South Bend)

One unacceptable solution was received.

Three late solutions to Problem 1 were received which were at least partially correct.

Undergraduate: Shyan Jeng Ho (EE)

Graduate: Ashish Rao (EE)

Other: Dan Vanderhan (St. Joseph's H.S., South Bend)