PROBLEM OF THE WEEK
Solution of Problem No. 3 (Fall 2001 Series)

Problem: Show that $5x \leq 8 \sin x - \sin 2x \leq 6x$ for $0 \leq x \leq \dfrac{\pi}{3}$.

Solution (by Steven Landy, Fac. Physics at IUPUI)

Let

$$
\begin{aligned}
f(x) &= 8 \sin x - \sin 2x \\
f'(x) &= 8 \cos x - 2 \cos 2x \\
f''(x) &= -8 \sin x + 4 \sin 2x = -8 \sin x (1 - \cos x).
\end{aligned}
$$

From these we see $f'(0) = 6$, $f'(\pi/3) = 5$, $f(0) = 0$, $f''(x) \leq 0$ on $[0, \pi/3]$.
Therefore

$$ 5 \leq f'(x) \leq 6 \quad \text{on} \quad [0, \pi/3]. $$

Integrating from 0 to x gives

$$ 5x \leq f(x) \leq 6x \quad \text{on} \quad [0, \pi/3]. $$

Also solved by:

Undergraduates: Damir Dzhafarov (Fr. MA), Haizhi Lin (MA), Lue Wei Lu (Sr. EE), Stevie Schraudner (Sr. CS/MA), Eric Tkaczyk (Jr. EE/MA)

Graduates: George Hassapis (MA), John Hunter (MA), Dave Keshavdas (EE), Jaehong Kim (MA), Sravanthi Konduri (CE), Chris Lomont (MA), Ashish Rao (EE), B. N. Reddy Vanga (Nucl E), K. H. Sarma (Nucl E), Amit Shirsat (CS), D. Subramanian & P. Ghosh (CHME)

Others: Michael Hamburg (Sr. St. Joseph's H.S., South Bend)

One unacceptable solution was received.

Correction: Mr. Rice's class at East Tipp Middle School should have been listed among the solvers of Problem 1.