PROBLEM OF THE WEEK
Solution of Problem No. 9 (Fall 2001 Series)

Problem: Determine, with proof, all the real-valued differentiable functions f, defined for real $x > 0$, which satisfy $f(x) + f(y) = f(xy)$ for all $x, y > 0$.

Solution (by Brahma N. R. Vanga, Gr. Nucl. Eng., edited by the Panel)

Differentiation w.r.t. x and then w.r.t. y gives

$$f'(x) = yf'(xy), \quad f'(y) = xf'(x, y),$$

hence

$$xf'(x) = yf'(y) \qquad \forall x, y > 0,$$

so

$$xf'(x) = c \text{ (constant)}.$$

Integration gives $f(x) = c \ln x + C$, but since $f(1) + f(1) = f(1)$, $f(1) = 0$, so $C = 0$. The general solution is

$$f(x) = c \ln x, \qquad c \in \mathbb{R}.$$

Also solved by:

Undergraduates: Damir Dzhafarov (Fr. MA), Haizhi Lin (Jr. MA), Gregg Sutton (Fr. Sci.)

Graduates: Danlei Chen (CHME), Keshavdas Dave (EE), Gajath Gunatillake (MA), George Hassapis (MA), Sravanthi Konduri (CE), A. Mangasuli (MA), Ashish Rao (EE), Amit Shirsat (CS), D. Subramanian & P. Ghosh (CHME)

Faculty: Steven Landy (Phys. at IUPUI)

Others: Jayprakash Chipalkatti (U.B.C. Canada), Donald Dichmann (Calif.), Jing Shao (Gr. So. China Tech.)

One unacceptable solution was received.