PROBLEM OF THE WEEK
Solution of Problem No. 10 (Fall 2001 Series)

Problem: Given a triangle $\triangle ABC$ and a point S inside, show that, if the areas of triangles $\triangle ABS, \triangle BCS, \triangle CAS$ are equal, then S is the centroid of $\triangle ABC$.

Solution (by Steven Landy, Fac. Physics at IUPUI, edited by the Panel)

Let $\overline{AS}, \overline{BS}, \overline{CS}$ be extended to intersect $\overline{BC}, \overline{AC}, \overline{AB}$ in A', B', C', resp. Let $\langle ABC\rangle$ denote the area of $\triangle ABC$, similarly for other triangles. Let q denote the common area of $\triangle ASB, \triangle BSC, \triangle CSA$. Now

$$\frac{\langle ASC'\rangle}{\langle BSC'\rangle} = \frac{|AC'|}{|BC'|}, \quad \text{the triangles have the same height}$$
$$\frac{\langle ACC'\rangle}{\langle BCC'\rangle} = \frac{|AC'|}{|BC'|}, \quad \text{the triangles have the same height.}$$

So

$$\frac{\langle ACC'\rangle}{\langle BCC'\rangle} = \frac{\langle ASC'\rangle + q}{\langle BSC'\rangle + q} = \frac{\langle ASC'\rangle}{\langle BSC'\rangle};$$
$$1 + \frac{q}{\langle BSC'\rangle} = \frac{\langle BSC'\rangle + q}{\langle BSC'\rangle} = \frac{\langle ASC'\rangle + q}{\langle ASC'\rangle} = 1 + \frac{q}{\langle ASC'\rangle}$$

which implies $\langle ASC'\rangle = \langle BSC'\rangle$, then $|AC'| = |BC'|$, so $\overline{CC}'$ is a median of $\triangle ABC$. So are $\overline{AA}', \overline{BB}'$, S is the intersection of the medians, S is the centroid.

Also solved by:

Undergraduates: Stevie Schraudner (Sr. CS/MA), Eric Tkaczyk (Jr. EE/MA)

Graduates: Ali R. Butt (ECE), Keshavdas Dave (EE), Gajath Gunatillake (MA), George Hassapis (MA), Ashish Rao (EE), Amit Shirsat (CS), D. Subramanian & P. Ghosh (CHME)

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Two unacceptable solutions were received.