

PROBLEM OF THE WEEK  
Solution of Problem No. 12 (Fall 2001 Series)

**Problem:** Evaluate  $\int_0^\pi \frac{\cos 4x - \cos 4\alpha}{\cos x - \cos \alpha} dx$ .

**Solution** (by the Panel)

$$\begin{aligned}\cos 4x - \cos 4\alpha &= 2\cos^2 2x - 1 - 2\cos^2 2\alpha + 1 \\ &= 2(\cos 2x + \cos 2\alpha)(\cos 2x - \cos 2\alpha) \\ &= 4(\cos 2x + \cos 2\alpha)(\cos x + \cos \alpha)(\cos x - \cos \alpha).\end{aligned}$$

$$\begin{aligned}I &= \int_0^\pi \frac{\cos 4x - \cos 4\alpha}{\cos x - \cos \alpha} dx = 4 \int_0^\pi (\cos 2x + \cos 2\alpha)(\cos x + \cos \alpha) dx \\ &= 4 \int_0^\pi (\cos 2x \cos x + \cos 2x \cos \alpha + \cos 2\alpha \cos x + \cos 2\alpha \cos \alpha) dx \\ &= 4 \int_0^\pi \left( \frac{1}{2}(\cos 3x + \cos x) + \cos 2x \cos \alpha + \cos 2\alpha \cos x + \cos 2\alpha \cos \alpha \right) dx.\end{aligned}$$

Since

$$\int_0^\pi \cos kx dx = 0 \quad (k = 1, 2, 3, \dots),$$

consequently

$$I = 4 \cos 2\alpha \cos \alpha.$$

Solved by:

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One incorrect solution was received. We received a correct late solution of Problem 11 from Rob Pratt (U. North Carolina).