PROBLEM OF THE WEEK
Solution of Problem No. 2 (Fall 2002 Series)

Problem: Suppose a mass moves smoothly enough along the x-axis. It starts at $x = 0$ with zero velocity and zero acceleration. At time $t = 2$ it reaches $x = a > 0$ with zero velocity and zero acceleration. Show that at some time between 0 and 2 the rate of change of acceleration is $\geq 3a$.

Solution (by Steven Landy, Fac. Physics at IUPUI, edited by the Panel)

The mass must move from $x = 0$ to $x = \dfrac{1}{2} a$ in time $t \leq 1$, or from $x = \dfrac{1}{2} x$ to $x = a$ in time $t \leq 1$. Consider the first case and let $\max\limits_{0 \leq t \leq 1} (\dddot{x}(t)) = m$. Then for $0 \leq t \leq 1, \quad \ddot{x}(t) \leq mt,$

$\dot{x}(t) \leq \dfrac{m}{2} t^2, \quad x(t) \leq \dfrac{m}{6} t^3, \quad \text{thus } \dfrac{a}{2} \leq \dfrac{m}{6} 1^3, \quad m \geq 3a.$ The second case works the same.

Also solved by:

Undergraduates: Jason Anderson (Fr. ME), Eric Tkaczyk (Sr. MA/EE)

Others: J.L.C. (Fishers, IN),

Three incorrect solutions were received.