## PROBLEM OF THE WEEK Solution of Problem No. 7 (Fall 2002 Series)

**Problem:** Given the equation  $x^n + a_1 x^{n-1} + a_2 x^{n-2} + \cdots + a_n = 0$  with real coefficients and  $a_1^2 < a_2$ . Show that not all the roots are real.

**Solution** (by the Panel, somewhat simpler than the submitted solutions) If  $a_1^2 < a_2$  then  $a_1^2 < 2a_2$ . But if  $r_i (i = 1, ..., n)$  are the roots of the given equation then

$$a_1^2 - 2a_2 = (-\sum r_i)^2 - 2\sum_{i < j} r_i r_j = \sum r_i^2$$

and  $\sum r_i^2 < 0$  cannot be true if all  $r_i$  are real.

Solved by:

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One incorrect solution was received.

Jia Han Li (ECE) submitted a late correct solution of Problem 6.