PROBLEM OF THE WEEK
Solution of Problem No. 9 (Fall 2002 Series)

Problem: Determine $\lim_{n\to\infty} \sum_{i=0}^{n} \sum_{j=0}^{n-i} \frac{x^j}{i!j!}$.

Solution (by Rob Pratt, Gr. Univ. of North Carolina)

$$\begin{aligned}
\lim_{n\to\infty} \sum_{i=0}^{n} \sum_{j=0}^{n-i} \frac{x^j}{i!j!} &= \lim_{n\to\infty} \sum_{i=0}^{n} \sum_{k=i}^{n} \frac{x^{k-i}}{i!(k-i)!} \quad \text{(change of index } k = i + j\text{)} \\
&= \lim_{n\to\infty} \sum_{k=0}^{n} \sum_{i=0}^{k} \frac{x^{k-i}}{i!(k-i)!} \quad \text{(interchange order of summation)} \\
&= \lim_{n\to\infty} \sum_{k=0}^{n} \frac{1}{k!} \sum_{i=0}^{k} \binom{k}{i} 1^i x^{k-i} \\
&= \lim_{n\to\infty} \sum_{k=0}^{n} \frac{1}{k!} (1+x)^k \quad \text{(binomial theorem)} \\
&= \sum_{k=0}^{\infty} \frac{(1+x)^k}{k!} \\
&= e^{1+x}
\end{aligned}$$

Also solved by:

Undergraduates: Mohd Z.A.Z. Abidin (So. Engr), Jason Andersson (Fr. MA)

Graduates: Chris Lomont (MA), YiHuang Shen (MA), Qi Xu (ChE)

Faculty: Steven Landy (Physics at IUPUI)

Others: J.L.C. (Fishers, IN), Luis Gonzales Sánchez (MA, Un. de Tafira, Canaries), Yuichi Yamane (Gr. MA, Fukuoka U., Japan)

Four solutions were unacceptable because of faulty reasoning.

We received a number of late solutions. To be on time a solution must be in our mailbox by noon on Tuesday. Please allow for a delay in the postal service. Late solutions of Problem 8 were received from:
Undergraduates: Mohd Z.A.Z. Abidin (So. Engr), Jason Andersson (Fr. MA), Patrick McCormick (Jr. A&AE), Mark Rempala (Sr. Chem), Ratna Santoso (Jr. CS)
Graduates: Tom Engelsman (ECE), Ashish Rao (EE), Amit Shirsat (CS)
Others: Dane Brooke (Boeing), Rob Pratt (UNC, Chapel Hill, NC)