PROBLEM OF THE WEEK

Solution of Problem No. 10 (Fall 2002 Series)

Problem: Given a triangle T with points A, B, C, one on the interior of each side, let Γ be the circle passing through A, B and C. Show that Γ is not smaller than the incircle of T.

Solution (by Yifau Liang, Gr. ECE)

Let d_a, d_b, d_c denote the distances of the center O of Γ from the sides a, b, c resp., let r denote the radius of Γ, ρ the radius of the incircle. Clearly,

$$d_a, d_b, d_c \leq r.$$

The area of T is given by

$$|T| = \frac{1}{2}(ad_a + bd_b + cd_c) \leq \frac{1}{2}(a+b+c)r$$

if O is inside T. Otherwise, there are one or two minus signs in the first sum but the upper bound remains the same. But also

$$|T| = \frac{1}{2}(a+b+c)\rho.$$

Hence $\rho \leq r$.

Also solved by:

<u>Faculty</u>: Steven Landy (Physics at IUPUI)

Correct late solutions were received from Eric Tkaczyk (Sr. EE/MA) and George Hassapis (Gr. MA)

One incorrect late solution was received.