PROBLEM OF THE WEEK
Solution of Problem No. 11 (Fall 2002 Series)

Problem: Let f be a function from the euclidean plane $\mathbb{R}^2$ to $\mathbb{R}$ with the property: if A, B, C are the vertices of any triangle in $\mathbb{R}^2$, with circumcenter O, then $\frac{1}{3}[f(A)+f(B)+f(C)] = f(O)$. Show that f is constant.

Solution (by Steven Landy, IUPUI Phys Staff)

Consider any two points in the plane P and Q. Draw any circle thru P and Q. Let O be its center and A and B two other points on the circle. Then by the given

$$\begin{aligned} f(O) &= \frac{1}{3}(f(A)+f(B)+f(P)) \\ &= \frac{1}{3}(f(A)+f(B)+f(Q)) \\ \therefore f(P) &= f(Q). \end{aligned}$$

Thus f is constant.

Also solved by:

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Others: J.L.C. (Fishers, IN), Luis Gonzales Sánchez (MA, Un. de Tafira, Canaries), Kishin Sadarangani (MA, Univ of Tafira, Las Palmas, Canaries), Leo Sheck (U. Auckland Med Sch, N.Z.)

The Octoberbreak confused the panel. We published solutions of Problems 7, 8, 9 a week early. This caused us to declare many of your solutions late though they were received on time. These will be counted as on time. Apologies.