

PROBLEM OF THE WEEK

Solution of Problem No. 10 (Fall 2003 Series)

Problem: The Lucas numbers are defined as $L_0 = 2$, $L_1 = 1$, $L_{n+2} = L_{n+1} + L_n$ for $n \geq 0$. Find a closed form for the sum $\sum_{k=0}^n L_k^2$ in terms of the L_n .

Solution (by Trushal V. Chokshi, Soph. ECE)

Since $L_k = L_{k+1} - L_{k-1}$ for $k \geq 1$, we have

$$\begin{aligned}\sum_{k=0}^n L_k^2 &= L_0^2 + L_1(L_2 - L_0) + L_2(L_3 - L_1) + \cdots + L_n(L_{n+1} - L_{n-1}) \\ &= L_0^2 - L_1 L_0 + L_n L_{n+1} \\ &= L_n L_{n+1} + 2.\end{aligned}$$

Another solution gives the result $L_{2n+1} + 2 + (-1)^n$.

Also solved by:

Undergraduates: Michael Chun Chang (So. Bio/Chem), Kedar Hippalgaonkar (Fr. ME), Jignesh V. Mehta (So. Phys)

Graduates: Fredy Aquino (Phys), George Hassapis (MA), Amit Shirsat (CS), Shun Zhang (MA)

Faculty: Steven Landy (Physics at IUPUI)

Others: Gagan Tara Nanda (Sr. UC Berkeley), Rob Pratt (UNC, Chapel Hill), Luis Gonzales Sánchez (MA, U. of Las Palmas, Spain)