PROBLEM OF THE WEEK

Solution of Problem No. 11 (Fall 2003 Series)

Problem: Given a circle K with center O and radius 1, and two points A, B in the same plane, show that the locus of centroids of triangles ABC with C on K is a circle. Determine its center and radius.

Solution (by Brahma N. R. Vanga, Gr. Nucl. Eng.))

Let the coordinates of A, B and C be $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) w.r.t. to an origin at the center of circle K. The centroid of the $\triangle$ is given by

$$(x_c, y_c) = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right).$$

By virtue of C lying on the circle and satisfying $x_3^2 + y_3^2 = 1$, we have

$$\left[3\left(x_c - \frac{x_1 + x_2}{3}\right)\right]^2 + \left[3\left(y_c - \frac{y_1 + y_2}{3}\right)\right]^2 = 1.$$

Therefore the locus of (x_c, y_c) is a circle with center at

$$\left(\frac{x_1 + x_2}{3}, \frac{y_1 + y_2}{3}\right),$$

which is the centroid of $\triangle OAB$, and radius $\dfrac{1}{3}$.

Also solved by:

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One incorrect solution was received.