PROBLEM OF THE WEEK
Solution of Problem No. 2 (Fall 2004 Series)

Problem: Prove that if two chords of an ellipse bisect each other, they are diameters.

Solution (by Steven Landy, Phys at IUPUI, edited by the Panel)

A linear transformation transforms the ellipse into a circle, so we only need to show the statement is true for circles.

Suppose AB and CD bisect each other and O is the point of intersection. Then $AO^2 = CO^2$ or $AO = CO$. The diagonals of quadralateral $ABCD$ then bisect each other and are equal. Hence $ACBD$ is a rectangle. Thus angle $ACB = 90^\circ$ so AB is a diameter. Similarly CD is a diameter.

Also, at least partially solved by:

Undergraduates: Syed Hassan (Aero & Astro), Xufeng Wang (Fr. Eng.)

Graduates: K. H. Sarma (Phys)

Others: Georges Ghosn (Quebec), M. Rappaport (Worcester Yeshiva Acad.), Sanjiv (ECE, Waterloo)

There were 5 unacceptable solutions.