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# PROBLEM OF THE WEEK

9/27/05 due NOON 10/10/05

CAN YOU GIVE US A SOLUTION?

## Problem No. 6 (Fall 2005 Series)

Let  $\phi$  be the Euler function defined by  $\phi(1) = 1$ , and for any integer  $n > 1$ ,  $\phi(n)$  is the number of positive integers  $\leq n$  and relatively prime to  $n$ . Prove that for all real  $x \neq \pm 1$ .

$$\sum_{m=0}^{\infty} (-1)^m \phi(2m+1) \frac{x^{2m+1}}{1+x^{4m+2}} = \frac{|x-x^3|}{(1+x^2)^2}.$$

A panel in the Mathematics Department publishes a challenging problem once a week and invites college & pre-college students, faculty, and staff to submit solutions. The objective of this is to stimulate and cultivate interest in good mathematics, especially among younger students. Solutions are due within two weeks from the date of publication. They can be faxed to (765) 494-0548 or sent by campus or U.S. mail (no E-mail please) to:

PROBLEM OF THE WEEK, **8th Floor**, Math Sciences Bldg., Purdue Univ.,  
150 North University St., West Lafayette, IN 47907-2067

Solvers should include their name, address, and **status at the University or school**.

The names of those who submitted correct solutions will be posted in the Math. Library, along with the best solution. Every Purdue student who submits three or more correct solutions will receive a Certificate of Merit. A prize fund of \$150.00 will be distributed among the Purdue undergraduates who have contributed at least six correct solutions for the total fall 2005 series.