PROBLEM OF THE WEEK

Solution of Problem No. 5 (Fall 2006 Series)

Problem: Show that two parabolas with the same focus, and whose axes do not lie along the same line, intersect in exactly two points.

Solution (by Hoan Duong, San Antonio College faculty, edited by the Panel)

Let the two directrices be d_1 and d_2, and let the common focus be F. Let Q be an intersection of the two parabolas. Then $d(d_1, Q) = d(F, Q) = d(d_2, Q)$. Hence Q is the center of a circle passing through F, and having d_1, d_2 as tangent lines. Since there are exactly two circles with this property, there are exactly two intersections.

At least partially solved by:

Undergraduates: Alan Bernstein (Sr. ECE), Nate Orlow (So, Math)

Graduates: Miguel Hurtado (ECE)

Others: Yunting Gao (China), Georges Ghosn (Quebec), K. Jeevarajan (Sri Lanka), Steven Landy (IUPUI Physics staff)