PROBLEM OF THE WEEK

Solution of Problem No. 9 (Fall 2006 Series)

Problem:

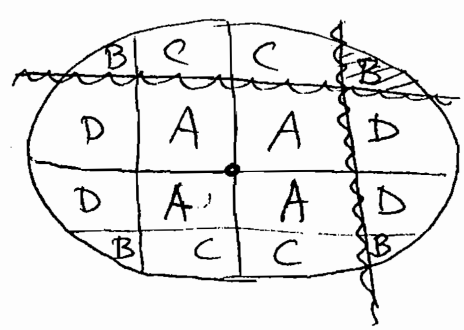
Let E be an ellipse with area 1. Given are two chords, parallel to the major and minor axes, respectively, that divide E into four regions. Prove that at least two of those regions have area not exceeding $\frac{1}{4}$.

Solution (by the Panel)

Without loss of generality, we may assume that the chords intersect in the first (closed) quadrant. Let us draw the axes of the ellipse, and chords symetric to the given ones about the minor and major axis, respectively. Then we get 16 subregions with areas as indicated on the diagram (some are allowed to be zero but they are all non–negative). Clearly, $B \leq \frac{1}{4}$. We will show that the sum of the areas of the two regions that have common side with the shaded one, is less or equal to than $\frac{1}{2}$. Indeed,

$$(B+2C)+(2D+B)=2(B+C+D)=\frac{1}{2}(1-4A)\leq\frac{1}{2}.$$

So, at least one of the numbers $B+2C$ and $2D+B$ does not exceed $\frac{1}{4}$. This completes the proof.



Update on Problems 6 and 7:

Problem 6 was also solved by Georges Ghosn.
Problem 7 was also solved by Georges Ghosn and Steven Landy.

At least partially solved by:

<u>Undergraduates</u>: Immanuel Alexander (So. MA&CS), Alan Bernstein (Sr. ECE), Nate Orlow (So, MA), Michael Snow (Jr. ME)

<u>Graduates</u>: Tom Engelsman (ECE)

<u>Others</u>: Yunting Gao (China), Georges Ghosn (Quebec), Steven Landy (IUPUI Physics staff)