PROBLEM OF THE WEEK
Solution of Problem No. 1 (Fall 2007 Series)

Problem: Let $d(n)$ denote the number of digits of n in its decimal representation. Evaluate the sum

$$\sum_{n=1}^{\infty} \frac{1}{d(n)!}.$$

Solution (by Alan Bernstein, Senior in ECE, Purdue)

Since $d(n) = k$ for $10^{k-1} \le n < 10^k$, the sum consists of
9 terms of the form $\frac{1}{1}$,
90 terms of the form $\frac{1}{2}$,
900 terms of the form $\frac{1}{6}$,
in general, $9 \cdot 10^{k-1}$ terms of the form $\frac{1}{k!}$.
Reindexing the sum,

$$\sum_{n=1}^{\infty} \frac{1}{d(n)!} = S = \sum_{n=1}^{\infty} \frac{9 \cdot 10^{n-1}}{(n)!}$$

$$\text{or} \quad S = \frac{9}{10} \sum_{n=1}^{\infty} \frac{10^n}{n!}$$

$$\text{or} \quad S = \frac{9}{10}(e^{10} - 1),$$

since $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + \sum_{n=1}^{\infty} \frac{x^n}{n!}$, and $e^x - 1 = \sum_{n=1}^{\infty} \frac{x^n}{n!}$.

Also solved by:

Undergraduates: Lokesh Batra (Fr. Engr), Noah Blach (Fr. Math), Ankit Kuwadekar (Fr. CS), Abram Magner (Fr, CS & Math), Siddharth Tekriwal (Fr. Engr.), Kevin Townsend (So, ECE), Kifer Christopher Troxell (Sr. Phys & Math)

Graduates: Tom Engelsman (ECE), Jim Vaught (ECE)

Others: Brian Bradie (Christopher Newport U. VA), Stephen Casey (Ireland), Matias V. Giusti (Sr. Univ. De Córdoba), Pete Kornya (Faculty, Ivy Tech), Steven Landy (IUPUI Physics), Camilo Montoya (Miami, FL), Angel Plaza (ULPGC, Spain), Sorin Rubinstein (PhD, TAU staff, Israel), Gearóid Ryan (Undergrad, Ireland)