PROBLEM OF THE WEEK

Solution of Problem No. 2 (Fall 2007 Series)

Problem: Let $3n$ points ($n \geq 2$) be distributed in space so that no 4 points are co–planar. Show that one can draw at least $3n^2$ line segments connecting these points without forming a tetrahedron (with vertices from the given points).

Solution (by Sorin Rubinstein, TAU staff, Israel)

Partition the points into three sets of n points each, and connect two points by a segment if and only if they do not belong to the same set.

Since among the $\binom{3n}{2}$ possible pairs of points, $3 \cdot \binom{n}{2}$ pairs have both components in the same set, there are:

$$\binom{3n}{2} - 3 \cdot \binom{n}{2} = \frac{3n(3n-1)}{2} - 3 \cdot \frac{n(n-1)}{2} = 3n^2$$

segments.

Among any 4 points at least two belong to the same set (because there are only three sets) and therefore are not connected by a segment. Thus the segments chosen this way answer the question.

Also solved by:

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<u>Graduates</u>: David Lomiashvili (Phys.)

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