

PROBLEM OF THE WEEK  
Solution of Problem No. 7 (Fall 2007 Series)

**Problem:** Determine the largest number  $d$  such that the following is true: If the points of the perimeter of an equilateral triangle of side 1 are colored with four colors, then there must be two points of the same color which are at least distance  $d$  apart.

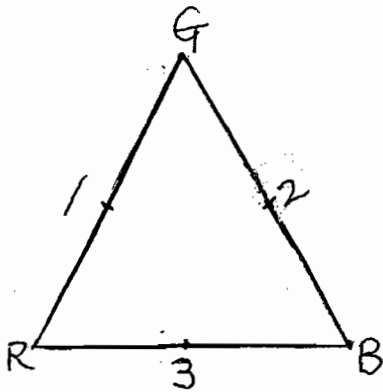
**Solution**

The answer is  $d = \frac{1}{2}$ , and two things must be proved:

1. The statement is true with  $d = \frac{1}{2}$ . (Proof taken from the solution submitted by David Lomiashvili, Purdue graduate student.)

Let us consider six points: 3 vertices and 3 midpoints. Distances between any two of them is not less than  $\frac{1}{2}$ . One can't color 6 points in such a way that any two of them have different color, which means that there are at least two points with same color.

2. The statement is false if  $d > \frac{1}{2}$ . (Proof taken from the solution submitted by Steven Landy, IUPUI physics faculty.)



R-red   B-blue   G-green  
Y-yellow  
points 1,2,3 are midpoints

If we color the intervals

$[1,R]$ $[R,3]$	red	then all pairs of points from the same color have distance $\leq \frac{1}{2}$ .
$[3,B]$ $[B,2]$	blue	
$[2,G]$ $[G,1]$	green	

The problem was solved by:

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