## PROBLEM OF THE WEEK

 Solution of Problem No. 8 (Fall 2007 Series)Problem: Two particles move in the plane so that their positions at time $t$ are $M_{t}=$ $(1+t, 1+t)$ and $N_{t}=(t-1,1-t)$. Let $\ell_{t}$ be the line through $M_{t}$ and $N_{t}$. Describe the set $S$ swept out by $\ell_{t}\left(\right.$ i.e., $\left.S=\bigcup_{t=-\infty}^{\infty} \ell_{t}\right)$.

Solution (by Hoan Duong, San Antonio College)
Since the slope of $l_{t}$ is $\frac{(1+t)-(1-t)}{(1+t)-(t-1)}=t$, an equation of the line $l_{t}$ is $y-(1+t)=$ $t[x-(1+t)]$.
Then

$$
\begin{aligned}
S & =\left\{(x, y) \mid(x, y) \in l_{t} \quad \text { for some } t \in R\right\} \\
& =\left\{(x, y) \mid t^{2}-t x+y-1=0 \quad \text { for some } t \in R\right\} \\
& =\left\{(x, y) \left\lvert\, t=\frac{x \pm \sqrt{x^{2}-4(y-1)}}{2} \in R\right.\right\} \\
& =\left\{(x, y) \mid x^{2}-4(y-1) \geq 0\right\} \\
& =\left\{(x, y) \left\lvert\, y \leq \frac{x^{2}}{4}+1\right.\right\} .
\end{aligned}
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Also solved by:

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