PROBLEM OF THE WEEK Solution of Problem No. 8 (Fall 2007 Series)

Problem: Two particles move in the plane so that their positions at time t are $M_t = (1 + t, 1 + t)$ and $N_t = (t - 1, 1 - t)$. Let ℓ_t be the line through M_t and N_t . Describe the set S swept out by ℓ_t (i.e., $S = \bigcup_{t=-\infty}^{\infty} \ell_t$).

Solution (by Hoan Duong, San Antonio College)

Since the slope of l_t is $\frac{(1+t)-(1-t)}{(1+t)-(t-1)} = t$, an equation of the line l_t is y - (1+t) = t[x - (1+t)]. Then

$$S = \{(x, y) | (x, y) \in l_t \text{ for some } t \in R\}$$

= $\{(x, y) | t^2 - tx + y - 1 = 0 \text{ for some } t \in R\}$
= $\left\{(x, y) \mid t = \frac{x \pm \sqrt{x^2 - 4(y - 1)}}{2} \in R\right\}$
= $\{(x, y) | x^2 - 4(y - 1) \ge 0\}$
= $\left\{(x, y) \mid y \le \frac{x^2}{4} + 1\right\}.$

Also solved by:

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