PROBLEM OF THE WEEK

Solution of Problem No. 2 (Fall 2008 Series)

Problem: Let p be a prime number. Show that $\binom{2p}{p} \equiv 2 \quad (\text{modulo} \quad p^2)$.

Solution (by Steve Spindler, Chicago)

Comparing the coefficients of X^p from the binomial expansions of $(1+X)^{2p} = (1+X)^p(1+X)^p$ yields:

$$\binom{2p}{p} = \sum_{k=0}^{p} \binom{p}{k}\binom{p}{p-k} = 2 + \sum_{k=1}^{p-1} \binom{p}{k}^2$$

Clearly, p does not divide $k!$ when $k < p$. Therefore, p divides $\binom{p}{k}$ for $1 < k < p$, and thus p^2 divides $\sum_{k=1}^{p-1} \binom{p}{k}^2 = \binom{2p}{p} - 2$.

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