PROBLEM OF THE WEEK

Solution of Problem No. 4 (Fall 2008 Series)

Problem: Let f be a real–valued function on $[0, \infty]$ such that

$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} f'''(x) = 0.$

Show that $\lim_{x\to\infty} f'(x) = \lim_{x\to\infty} f''(x) = 0.$

Solution (by Minghua Lin & Zhang Xiao, Shaanxi Normal University, China)

$\forall x \in [0, +\infty)$, we have

$$f(x+1) = f(x) + f'(x) + \frac{f''(x)}{2!} + \frac{f'''(\xi)}{3!}, \quad \xi \in (x, x+1) \qquad (1)$$

$$f(x+2) = f(x) + 2f'(x) + 2f''(x) + \frac{8f'''(\eta)}{3!}, \quad \eta \in (x, x+2) \qquad (2)$$

Let $x \to +\infty$, then $\xi \to +\infty$ and $\eta \to +\infty$.

From (1), we have $\lim_{x\to+\infty} \left[f'(x) + \frac{f''(x)}{2}\right] = 0 \qquad (3)$

From (2), we have $\lim_{x\to+\infty} [f'(x) + f''(x)] = 0 \qquad (4)$

From (3) – (4), we have $\lim_{x\to+\infty} f''(x) = 0$ and so $\lim_{x\to+\infty} f'(x) = 0$.

Also solved by:

<u>Graduates</u>: Britain Cox (Math)

<u>Others</u>: Hoan Duong (San Antonio College), Elie Ghosn (Montreal, Quebec), Steven Landy (IUPUI Physics staff), Sorin Rubinstein (TAU faculty, Israel)