PROBLEM OF THE WEEK
Solution of Problem No. 7 (Fall 2008 Series)

Problem: Let three points be chosen at random from the unit circle (independently uniformly distributed in the circle). What is the probability that the center of the circle is in the triangle whose vertices are the chosen points?

Solution (by Steven Landy, IUPUI Physics staff)

The center is in triangle ABC if and only if the points A, B, C do not lie in the same semi–circle. Without loss of generality, let $A = (1, 0)$ and B have $y > 0$. The center is in the triangle if C is in shaded region.
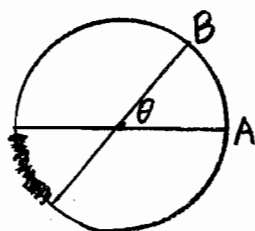
$$p = \frac{1}{\pi} \int_0^\pi \frac{\theta}{2\pi} \, d\theta = \frac{1}{2\pi^2} \frac{\pi^2}{2} = \frac{1}{4}$$

Also solved by:

Undergraduates: David Elden (So. Mech. Engr)

Graduates: Sambit Palit (ECE), Huanyu Shao (CS), Peter Weigel (Math)

Others: Manuel Barbero (New York), Kaushik Basu & Apurva Somani(Graduate student, Univ. of Minnesota, Twin Cities), Brian Bradie (Christopher Newport U. VA), Hoan Duong (San Antonio College), Subham Ghosh (Grad student, Washington Univ. St. Louis), Elie Ghosn (Montreal, Quebec), Sleiman Jradi (Freshman, Christopher Newport Univ.), Kevin Laster (Indianapolis, IN), Sorin Rubinstein (TAU faculty, Israel), Bill Wolber Jr. (ITaP)