## PROBLEM OF THE WEEK Solution of Problem No. 9 (Fall 2008 Series)

**Problem:** Suppose the unit square is divided into four parts by two perpendicular lines, each parallel to an edge. Show that at least two of the parts have area no larger than  $\frac{1}{4}$ .

Solution (by David Elden, Sophomore, Mech. Engineering)

Choose a coordinate system such that the vertices of the square are located at (0,0), (1,0), (0,1), and (1,1). Let the lines x = a and y = b describe the cuts. Further assume, without loss of generality, that  $a \le 1/2$ ,  $b \le 1/2$ , and  $a \le b$ .

Then the area of the region bounded by the lines x = 0, y = 0, x = a, and y = b has an area of (a - 0)(b - 0) = ab, which has a maximum value of (1/2)(1/2) = 1/4.

Also, the area of the region bounded by the lines x = 0, x = a, y = b, and y = 1 is

$$A_2 = (a - 0)(1 - b) = a(1 - b) = a - ab$$

Since  $b \ge a$ , we are assured that  $A_2 \le a - a^2$ . This function has a maximum value of 1/4 over the interval  $a \in [0, 1/2]$ .

Also solved by:

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