PROBLEM OF THE WEEK
Solution of Problem No. 10 (Fall 2009 Series)

Problem: Prove or disprove: There is at least one straight line normal (perpendicular) to the graph of $y = \cosh x$ at a point $(a, \cosh a)$ and also normal to the graph of $y = \sinh x$ at a point $(b, \sinh b)$

Solution (by Andy Bohn, Sr. Physics, Purdue University)

The perpendicular to $y = \cosh x$ at $x = a$ has slope $\dfrac{-1}{\sinh a}$. The perpendicular to $y = \sinh x$ at $x = b$ has slope $\dfrac{-1}{\cosh b}$. Therefore the normal line equations are
From cosh:

$$\sinh a[y - \cosh a] + [x - a] = 0.$$

From sinh:

$$\cosh b[y - \sinh b] + [x - b] = 0.$$

For these lines to coincide, their slopes must be equal, or $\sinh a = \cosh b$. Also:

$$a + \sinh a \cosh a = b + \cosh b \sinh b.$$

So

$$b - a = \sinh a \cosh a - \cosh b \sinh b = \cosh b \cosh a - \sinh a \sinh b = \cosh(b - a).$$

But $\cosh(b - a) > b - a$ always, so there cannot be such a line.

The problem was also solved by:

Undergraduates: Clara Bennett (Phys), Kilian Cooley (Fr.), Kun–Chieh Wang (Sr. Math)

Graduates: Richard Eden (Math), Rodrigo Ferraz de Andrade (Math), Gabriel Sosa (Math), Tairan Yuwen (Chemistry)

Others: Andrea Altamura (Italy), Manuel Barbero (New York), Brian Bradie (Christopher Newport U. VA), Gruian Cornel (IT, Romania), Tom Engelsman (Chicago, IL), Elie Ghosn (Montreal, Quebec), Jeffery Hein (CS & Math, Purdue Univ. Calumet), Steven Landy (IUPUI Physics staff), Kevin Laster (Indianapolis, IN), Wei-hsiang Lien (Grad student, National Chiao-Tung Univ., Taiwan), Angel Plaza (ULPGC, Spain), Sorin Rubinstein (TAU faculty, Israel), Craig Schroeder (Grad student, Stanford Univ.) Yansong Xu (Brandon, FL), Thierry Zell (Ph.D, Purdue 03)