

PROBLEM OF THE WEEK
Solution of Problem No. 13 (Fall 2009 Series)

Problem: Show how to construct the radius of an impenetrable spherical ball with compass, straight-edge, and a rigid flat surface. The compass can be used to draw circles either on the surface of the ball or on the flat surface and also to transfer distances (as measured along straight lines in space) from one surface to the other. The straight-edge can be used only to draw straight lines on the flat surface.

Solution (by Sorin Rubinstein, TAU staff, Israel)

We “center”* the compass at a point A of the ball and draw the circle C_1 on the ball. With the same opening of the compass we “center” the compass at a point B of the ball and draw the circle C_2 on the ball. The point B is so chosen that the circles C_1 and C_2 intersect each other. Let P and Q be the intersection points of C_1 and C_2 . The intersection of the perpendicular bisector plane of the segment AB with the ball’s surface is a great circle C . Moreover, since C_2 and C_1 are symmetric to each other with respect to the reflection in the perpendicular bisector plane of the segment AB , the points P and Q belong to C . Now, we change and fix the opening of the compass such that the circles drawn on the sphere with this opening and with the compass “centered” at A and B respectively intersect at two new points R and S . Then P, Q, R and S belong to the same great circle C . We can now construct on the flat surface a triangle $P'Q'R'$ such that $|P'Q'|$ equals the distance between P and Q , $|Q'R'|$ equals the distance between Q and R and $|P'R'|$ equals the distance between P and R . By standard plane geometry we construct the circumcenter O' of the triangle $P'Q'R'$. Then $O'P'$ is the radius of the given ball.

* By centering the compass at a point A we mean that we fasten the tip of the spike of the compass at the point A - which, of course, is not the center of the circle drawn on the ball.

The problem was also completely or partially proved by:

Undergraduates: Artyom Melanich (Fr. Engr.)

Graduates: Tairan Yuwen (Chemistry)

Others: Andrea Altamura (Italy), Gruian Cornel (IT, Romania), Elie Ghosn (Montreal, Quebec), Steven Landy (IUPUI Physics staff), Craig Schroeder (Grad student, Stanford Univ.)