PROBLEM OF THE WEEK
Solution of Problem No. 1 (Fall 2010 Series)

**Problem:** Let $A$ be any set of 39 distinct integers chosen from the arithmetic progression $6, 33, 60, \ldots, 1977$.

Prove that there must be two distinct integers in $A$ whose sum is 2010.

**Solution** (by Eric Haengel, Junior, Physics/Math)

The numbers $6, 33, 60, \ldots, 1977$ are all of the form $6 + 27n$ where $n \in A = \{0, \ldots, 73\}$. If $n, m \in A$ such that $n + m = 74$, then

$$(6 + 27n) + (6 + 27m) = 12 + 27 \cdot 74 = 2010.$$

So it suffices to show that any collection of 39 distinct integers in $A$ will contain two numbers that add up to 74. Suppose the contrary: there exists a subset $B$ of $A$ containing 39 integers, such that no two add up to 74.

Thus, if $n \in B$, $(74 - n) \notin B$. This means that $B$ cannot contain both numbers in the pairs $(1, 73), (2, 72), \ldots, (36, 38)$. Apart from these, $B$ may contain 0 and 36, and counting it all up, $B$ can contain at most $1 + 1 + 36 = 38$ elements, which is a contradiction.

The problem was also solved by:

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