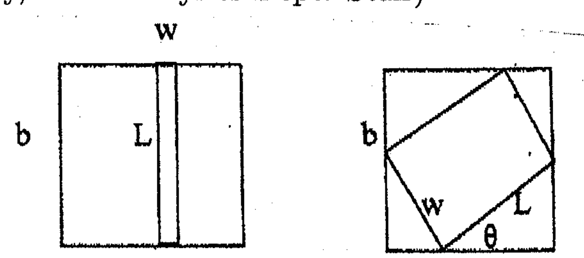
PROBLEM OF THE WEEK

Solution of Problem No. 12 (Fall 2010 Series)

Problem: For $0 < w < \sqrt{2}B$, let L be the largest number such that some $L \times w$ rectangle R is contained in a square S of edge length B. You may assume that the maximal rectangle R is inscribed in S; i.e., that each vertex of R is on the boundary of S. Calculate $L = L(w, B)$ and in particular determine when $L > B$.

**This problem is a modified version of one proposed by Michael Roach of New Millemium Building Systems, who has an engineering application for it.

Solution: (by Steven Landy, IUPUI Physics Dept. Staff)



There are two ways the rectangle can be inscribed.

Case I. The rectangle touches two opposite sides of the square. In this case we have $w \le b$ and $L = b$.

Case II. The rectangle touches all four sides. The following equations are evident from the picture.

$$L\cos\theta + w\sin\theta = b$$
$$w\cos\theta + L\sin\theta = b.$$

Combining these we get $(L - w)(\sin\theta - \cos\theta) = 0$. So there are two subcases.

a. $L = w$. If $L = w$, the rectangle is a square. This square is interior to the square of side b, and so $w \le b$ and $L \le b$. Since for $w \le b$ case I yields $L = b$, subcase a is irrelevant.

b. $\theta = 45°$. Plugging into the above, $L + w = b\sqrt{2}$ or, $L = b\sqrt{2} - w$. This case may be used for any w between 0 and $b\sqrt{2}$.

We should use case I if $b > b\sqrt{2} - w$. That is when $w > b\sqrt{2} - b$. Recall for case I, $w \le b$. In the special situation $w = b$, case I gives $L = b$, and case II gives $L = b\sqrt{2} - b$, so case I should apply.

So our function is this

$$L(w,b) = b \quad \text{if} \quad b\sqrt{2} - b \le w \le b$$
$$L(w,b) = b\sqrt{2} - w \quad \text{otherwise.}$$
$$L > b \qquad \text{for} \quad w < b\sqrt{2} - b.$$

Also completely or partially solved by:

Undergraduates: Jorge Ramos (So. Phys)

Graduates: Tairan Yuwen (Chemistry)

Others: Al-Sharif Talal Al-Housseiny (Grad. student, Princeton Univ.), Gruian Cornel (IT, Romania), Elie Ghosn (Montreal, Quebec), Sorin Rubinstein (TAU faculty, Israel), Steve Spindler (Chicago), Stephen Taylor (Bloomberg L.P. NY)