PROBLEM OF THE WEEK
Solution of Problem No. 3 (Fall 2010 Series)

Problem: Let n be any positive integer, and $d_1, \ldots, d_k$ the set of all positive integer divisors (including 1 and n) of n.
Show that $d_1 d_2 \ldots d_k = n^{k/2}$.

Solution (by Han Liu, Freshman, Math major)

For any one of $d_1, d_2, \ldots, d_k$, $\frac{n}{d_i}$ is also an integer, as d_i is a divisor of n. Thus $\frac{n}{d_i}$ is also a divisor of n; $\frac{n}{d_i}$ must be one of $d_1, d_2, \ldots, d_k$.
As $d_1, d_2, \ldots, d_k$ are all distinct, $\frac{n}{d_1}, \frac{n}{d_2}, \ldots \frac{n}{d_k}$ are all distinct.
Therefore $\frac{n}{d_1}, \frac{n}{d_2}, \ldots, \frac{n}{d_k}$ is just a rearrangement of $d_1, d_2, \ldots, d_k$. Thus $(d_1 \cdot \frac{n}{d_1})$
$(d_2 \cdot \frac{n}{d_2}) \ldots (d_k \cdot \frac{n}{d_k}) = n^k = (d_1 d_2 \ldots d_k)^2$.
Thus $d_1 d_2 \ldots d_k = n^{\frac{k}{2}}$.

The problem was also solved by:

Undergraduates: Ka Wang Chow (Sr. Science), Kilian Cooley (So.), David Elden (Sr. Mech. Engr), Eric Haengel (Jr. Math & Physics), Artyom Melanich (So. Engr.), Anurag Somani (Fr. Phys), Yixin Wang (So.)

Graduates: Jyotishka Datta (Stat.), Richard Eden (Math), Rodrigo Ferraz de Andrade (Math), Karthikeyan Marimuthu (Grad Student) Benjamin Philabaum (Phys.), Krishnaraj Sambath (Ch.E.), Huanyu Shao (CS), Tairan Yuwen (Chemistry)

Others: Neacsu Adrian (Romania), Siavash Ameli (Grad. student, Toosi Univ. of Tech, Iran), Syd Amit (Graduate student, Boston College), Manuel Barbero (New York), Hongwei Chen (Christopher Newport U. VA), Gruian Cornel (IT, Romania), Sandipan Dey (Graduate student, UMBC), Tom Engelsman (Chicago, IL), Nathan Faber (CO), Elie Ghosn (Montreal, Quebec), Boughanmi Mohamed Hedi, Swami Iyer (U. Massachusetts, CS), Steven Landy (IUPUI Physics staff), Kevin Laster (Indianapolis, IN), Wei-hsiang Lien (Research assistant, National Chiao-Tung Univ., Taiwan), Amitay Nachmani, Louis Rogliano (Corsica), Sorin Rubinstein (TAU faculty, Israel) Craig Schroeder (Ph.D. student, Stanford Univ.), Mark Sellke (Harrison High School, IN), Dong-Gil Shin (12th grade, Newton North HS), Steve Spindler (Chicago), Benjamin Tsai, Sahana Vasudevan (Palo Alto HS, CA), Turkay Yolcu (Visiting at Purdue U.), Kathy Zhong