

PROBLEM OF THE WEEK
Solution of Problem No. 9 (Fall 2010 Series)

Problem: Let a, b be real numbers, and let $0, u_1, u_2, \dots$ be a sequence satisfying $u_{n+1} = au_n + bu_{n-1}$, $n \geq 1$. Show that $f(x) = \sum_{n=1}^{\infty} u_n \frac{x^n}{n!}$ satisfies $f(x) = -e^{ax} f(-x)$.

Solution (by Peter Weigel, Graduate student, Purdue University)

$f(x)$ is the Taylor series of the unique solution to

$$\begin{aligned}y'' - ay' - by &= 0 \\ y(0) &= 0 \quad y'(0) = u_1.\end{aligned}$$

$-e^{ax} f(-x)$ also solves the initial value problem. Hence the two are equal.

Also completely or partially solved by:

Undergraduates: Kilian Cooley (So.), Eric Haengel (Jr. Math & Physics), Han Liu (Fr. Math), Artyom Melanich (So. Engr.), Yue Pu (Fr. Exchanged student), Yixin Wang (So.)

Graduates: Shuhao Cao (Math), Krishnaraj Sambath (Ch.E.), Tairan Yuwen (Chemistry)

Others: Neacsu Adrian (Romania), Siavash Ameli (Grad. student, Toosi Univ. of Tech, Iran), Manuel Barbero (New York), Hongwei Chen (Christopher Newport U. VA), Gruian Cornel (IT, Romania), Tom Engelsman (Chicago, IL), Elie Ghosn (Montreal, Quebec), D. Kipp Johnson (Teacher, Valley Catholic School, OR), Steven Landy (IUPUI Physics staff), Denes Molnar (Physics, Assistant Professor), Louis Rogliano (Corsica), Sorin Rubinstein (TAU faculty, Israel), Craig Schroeder (Ph.D. student, Stanford Univ.), Jason L. Smith (Professor, Phys. & Math. Richland Community College), Stephen Taylor (Bloomberg L.P. NY), Henri Vullierme (Universite Paris VI, France)