PROBLEM OF THE WEEK Solution of Problem No. 10 (Fall 2011 Series)

Problem: Are there positive irrational numbers a and b such that a^b is rational?

Solution: (by Charles Christoffer, Freshman, Engineering, Purdue University)

Yes. Consider the number $\sqrt{2}^{\sqrt{2}}$. I do not know whether it is rational or irrational, but I do know by application of the rational root theorem to $p(x) = x^2 - 2$ that $\sqrt{2}$ is irrational.

If $\sqrt{2}^{\sqrt{2}}$ is rational, than both a and $b = \sqrt{2}$.

If, on the other hand, $\sqrt{2}^{\sqrt{2}}$ is irrational, then setting $a = \sqrt{2}^{\sqrt{2}}$ and $b = \sqrt{2}$ yields $a^b = (\sqrt{2}^{\sqrt{2}})^{\sqrt{2}} = \sqrt{2^2} = 2$, which is an integer and therefore rational.

Either way, QED.

The problem was also solved by:

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