## PROBLEM OF THE WEEK

Solution of Problem No. 10 (Fall 2011 Series)

Problem: Are there positive irrational numbers $a$ and $b$ such that $a^{b}$ is rational?

Solution: (by Charles Christoffer, Freshman, Engineering, Purdue University)

Yes. Consider the number $\sqrt{2}^{\sqrt{2}}$. I do not know whether it is rational or irrational, but I do know by application of the rational root theorem to $p(x)=x^{2}-2$ that $\sqrt{2}$ is irrational.

If $\sqrt{2}^{\sqrt{2}}$ is rational, than both $a$ and $b=\sqrt{2}$.
If, on the other hand, $\sqrt{2}^{\sqrt{2}}$ is irrational, then setting $a=\sqrt{2}^{\sqrt{2}}$ and $b=\sqrt{2}$ yields $a^{b}=\left(\sqrt{2}^{\sqrt{2}}\right)^{\sqrt{2}}=\sqrt{2^{2}}=2$, which is an integer and therefore rational.

Either way, QED.

## The problem was also solved by:

Undergraduates: Kilian Cooley (So.), Sean Fancher (Science), Kaibo Gong (Sr. Math), Sidharth Mudgal Sunil Kumar (Fr. Engr.), Bennett Marsh (Fr. Engr.), Yixin Wang (Jr. ECE)

Graduates: Vaibhav Gupta (ECE), Sambit Palit (ECE), Tairan Yuwen (Chemistry)

Others: Adil Assouab (Freshman, U. of Indianapolis), Manuel Barbero (New York), Mojtaba Biglari (U. of Teheran), Charles Burnette (Philadelphia), Nicolas Busca (France), Hongwei Chen (Faculty, Christopher Newport U. VA), Gruian Cornel (Cluj-Napoca, Romania), Hubert Desprez (Jussieu University, France), Tom Engelsman (Tampa, FL), Andrew Garmon (So, Phys. Christopher Newport Univ.), Elie Ghosn (Montreal, Quebec), Martin Kleinsteuber (Germany), Chris Kyriazis (High school teacher, Chalki, Greece), Steven Landy (IUPUI Physics staff), Kevin Laster (Indianapolis, IN), Matt Mistele (FL), Achim Roth (Data Protection Officer, Germany), Sorin Rubinstein (TAU faculty, Israel), Craig Schroeder (Postdoc. UCLA), Leo Sheck (Faculty, Univ. of Auckland), Jason L. Smith (Professor, Phys. \& Math. Richland Community College), Steve Spindler (Chicago), Siddhi Venkatraman (Mclean, VA), Henri Vullierme (Universite Paris VI, France), Yansong Xu (Germantown, MD)

