PROBLEM OF THE WEEK

Solution of Problem No. 7 (Fall 2011 Series)

Problem: For every integer $n \geq 2$, prove that

$$\sum_{k=1}^{n} (-1)^k k \binom{n}{k} = 0,$$

where $\binom{n}{k}$ is the usual binomial coefficient.

Solution: (by Hubert Desprez, Paris, France)

Let's consider $\varphi(x) = \sum_{q=0}^{n} \binom{n}{q} x^q = (1+x)^n$. We have

$$\varphi'(x) = \sum_{q=0}^{n} q \binom{n}{q} x^{q-1} = n(1+x)^{n-1},$$

which implies $-\sum_{q=0}^{n} q \binom{n}{q} (-1)^{q-1} = -\varphi'(-1) = 0.$

The problem was also solved by:

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