PROBLEM OF THE WEEK
Solution of Problem No. 8 (Fall 2011 Series)

Problem: A "polygon" means a closed plane figure with vertices and straight edges, with exactly two edges meeting at each vertex, and no two edges meeting (except at a vertex).

Show that, in every polygon with more than three edges, there must be two vertices A, B (not connected by any edge) such that the segment AB lies in the interior of the polygon and meets no edge of the plygon (except at A and B!).

Solution: (by Steven Landy, IUPUI Physics Staff)

Assume that there is no interior diagonal. Every polygon must have at least one convex vertex (where the internal angle is less than 180°.) Let A, B, C be consecutive vertices, B a convex vertex, and suppose AC is along the x axis and B above it. Then AC will be an interior diagonal unless another vertex of the polygon lies in the interior of triangle ABC or on AC. Of all these vertices, let D be one having the largest y coordinate. Then BD is an interior diagonal.

The problem was also solved by:

Undergraduates: Sidharth Mudgal Sunil Kumar (Fr. Engr.)

Graduates: Paul Farias (IE), Vaibhav Gupta (ECE), Tairan Yuwen (Chemistry)

Others: Manuel Barbero (New York), Charles Burnette (Philadelphia), Gruian Cornel (Romania), Hubert Desprez (Jussieu University, France), Elie Ghosn (Montreal, Quebec), Jae Woo Jeon (Seoul, Korea), Kevin Laster (Indianapolis, IN), Achim Roth (Data Protection Officer, Germany), Craig Schroeder (Postdoc. UCLA), Leo Sheck (Faculty, Univ. of Auckland)