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# PROBLEM OF THE WEEK

9/11/12 due NOON 9/24/12

CAN YOU GIVE US A SOLUTION?

## Problem No. 4 (Fall 2012 Series)

- (a) Prove that if  $f$  is continuous on  $[0, 1]$  and differentiable on  $(0, 1)$  and satisfies  $|f'(x)| \leq M$  for some positive number  $M$  then

$$\left| \int_0^1 f(x) dx - \sum_{k=0}^{n-1} f\left(\frac{k}{n}\right) \frac{1}{n} \right| \leq \frac{M}{n}$$

- (b) (optional; the problem will be counted as solved if part (a) is solved) Show that the  $\frac{M}{n}$  of part (a) can be improved to  $\frac{M}{2n}$ .

A panel in the Mathematics Department publishes a challenging problem once a week and invites college & pre-college students, faculty, and staff to submit solutions. The objective of this is to stimulate and cultivate interest in good mathematics, especially among younger students. Solutions are due within two weeks from the date of publication.

Solutions can be emailed only as a pdf attachment to: [sfchang@math.purdue.edu](mailto:sfchang@math.purdue.edu). Solutions can also be faxed to 765-496-3177 or sent by campus or U.S. mail to:

PROBLEM OF THE WEEK, **6th Floor**, Math Sciences Bldg., Purdue Univ.,  
150 North University St., West Lafayette, IN 47907-2067

Please include your name, address and **status at your university or school** on your problem solutions.

The names of those who submitted correct solutions will be posted on the Problem of the Week website and in the Math. Library, along with the best solution. Every Purdue student who submits three or more correct solutions will receive a Certificate of Merit. A prize fund of \$300.00 will be distributed among those Purdue undergraduates who have contributed at least six correct solutions for the thirteen problems in the Fall 2012 series.