PROBLEM OF THE WEEK

Solution of Problem No. 5 (Fall 2012 Series)

Problem:

Show that every set of $n+1$ integers chosen from $\{1, 2, \ldots, 2n\}$ contains a pair of integers such that one is a multiple of the other.

Solution: (by Steve Spindler, Chicago)

Consider the largest odd divisors of each of the $n+1$ integers. They form a set of $n+1$ odd integers between 1 and $2n$. But there are only n odd integers between 1 and $2n$, so by the pigeon–hole principle at least two of the largest odd divisors are equal. The smaller of the two original integers divides the larger, since they differ by only a factor of a power of two.

The problem was also solved by:

Undergraduates: Bennett Marsh (So. Engr.), Rustam Orazaliyev (Fr. Actuarial Sci), Yicun Qian (Jr. Math & Stat), Lirong Yuan (Jr. Math & CS)

Graduates: Jeremy Troisi (Stat), Tairan Yuwen (Chemistry)

Others: Manuel Barbero (New York), Radouan Boukharfane (Graduate student, Montreal, Canada), Hongwei Chen (Professor, Christopher Newport Univ., Virginia), Hubert Desprez (Paris, France), Tom Engelsman (Tampa, FL), Bruce Fleischer, Scott Huber (Phys. Graduate Student, Ohio State Univ.), Jae-woo Jeon (Seoul, South Korea), Kipp Johnson (Valley Catholic HS teacher, Oregon), Steven Landy (Physics Faculty, IUPUI), Kevin Laster (Indianapolis, IN), Matthew Lim, Xiaoyin Liu (So. Univ. of North Carolina), Sun Hong Rhie(Granger, IN), Achim Roth (Data Protection Officer, Germany), Sorin Rubinstein (TAU faculty,Tel Aviv, Israel), Craig Schroeder (Postdoc. UCLA), Patrick Soboleski (Math teacher, Zionsville Community HS) Man Cheung Tsui, UCLA Student, CA