PROBLEM OF THE WEEK Solution of Problem No. 7 (Fall 2012 Series)

Problem:

Find all functions f on the integers which satisfy f(k) > 0 for all k, f(0) = 9, and $f(k) = \frac{1}{2}(f(k+1) + f(k-1))$ for all k.

Solution: (by Steven Landy, Physics Faculty, IUPUI)

Find all functions f on the integers which satisfy (for all integers k)

$$f(k) > 0 \tag{1}$$

$$f(0) = 9 \tag{2}$$

$$f(k) = 1/2(f(k+1) + f(k-1))$$
(3)

(3) implies that a straight line passes through any three consecutive points (k, f(k)). Then a single line must pass thru all such points. (1) and (2) force the line to be horizontal thru (0,9). Thus there is a unique function f(k) = 9.

The problem was also solved by:

<u>Undergraduates</u>: Bennett Marsh (So. Engr.), Rustam Orazaliyev (Fr. Actuarial Sci), Scott Podlogar (Fr. Engr), Yicun Qian (Jr. Math & Stat), Ying Xu (So. Engr), Lirong Yuan (Jr. Math & CS), Zaiwei Zhang (So. EE)

Graduates: Krishnaraj Sambath (ChE), Tairan Yuwen (Chemistry)

<u>Others</u>: Issam Abdallah Saleh Aburub (Amman, Jordan), Pierre Castelli (Antibes, France), Hongwei Chen (Professor, Christopher Newport Univ., Virginia), Gruian Cornel (Cluj-Napoca, Romania), Robert Desjardins (HS student, Montreal, Canada), Tom Engelsman (Tampa, FL), Andrew Garmon (Sr, Phys. Christopher Newport Univ.), Kipp Johnson (Valley Catholic HS teacher, Oregon), Matthew Lim, Xiaoyin Liu (So. Univ. of North Carolina), Denes Molnar (Professor, Physics, Purdue Univ.), Jean Pierre Mutanguha (Student, Oklahoma Christian Univ.), Charles Roldan (BS Math 2010, Purdue), Achim Roth (Data Protection Officer, Germany), Craig Schroeder (Postdoc. UCLA), Jason L. Smith (Professor, Richland Community College, IL), Patrick Soboleski (Math teacher, Zionsville Community HS), Steve Spindler (Chicago), Man Tsui (UCLA Student, CA)