PROBLEM OF THE WEEK
Solution of Problem No. 1 (Fall 2014 Series)

**Problem:**
**Find, with proof, the smallest possible value for $1-\frac{1}{n}-\frac{1}{m}-\frac{1}{k}$, where $k$, $m$ and $n$ are three different positive integers and $\frac{1}{n}+\frac{1}{m}+\frac{1}{k}<1$.**

This problem was contributed by Michael M. Brady of Houston, Texas.

**Solution: (by Bennett Marsh, Junior, Physics & Math, Purdue University)**

The smallest possible value is $1-\frac{1}{2}-\frac{1}{3}-\frac{1}{7}=\frac{1}{42}$. To see that this is optimal, we first note that each of $n,m,k$ must be greater than one, and one of them, say $n$, must be a 2. Otherwise the minimum value would have to be $1-\frac{1}{3}-\frac{1}{4}-\frac{1}{5}=\frac{13}{60}$, worse than above. The problem is then reduced to minimizing $\frac{1}{2}-\frac{1}{m}-\frac{1}{k}$ subject to $\frac{1}{m}+\frac{1}{k}<\frac{1}{2}$. Similarly to above, we must have $m,k>2$, and one, say $m$ must be a 3. Otherwise, the smallest possible value would be $\frac{1}{2}-\frac{1}{4}-\frac{1}{5}=\frac{1}{20}$. All that is left then is to choose $k$ such that $\frac{1}{k}<\frac{1}{6}$. The best possible choice is clearly $k=7$.

**The problem was also solved by:**

Undergraduates: Eric Buedel (Fr. Math), Yuanzheng Cao (Fr.), Yucheng Chen (So, Engr.), Xiaoyu Feng (Sr. Math& Stats), Scott Kelly (Jr. CS), Sameer Manchanda (Fr. CS), Rustam Orazaliyev (Jr. Actuarial Sci), Nick Perkins (Fr. FYE), Bryan Rainey (Jr. CS)

Graduates: Langlei Fang (AAE), Ellen Weld (Math), Tairan Yuwen (Chemistry)

Others: Manuel Barbero (New York), KD Harald Bensom (Germany), Marco Biagini (Math Teacher, Italy), Charles Burnette (Grad Student, Drexel Univ.), Hongwei Chen (Professor, Christopher Newport Univ. Virginia), Dylan Cordaro (Don Bosco Prep), Hubert Desprez (Paris, France), Sandipan Dey (UMBC Alumni), Tom Engelsman (Tampa, FL), Ghasem Esmati (Sharif Univ. of Tech), Talal AL Fares (Lebanon), Paul Farias (W. Lafayette, IN), Andrew Garmon (Christopher Newport University alumni), Rick Shilling & Bruce Grayson (Orlando, FL), Aaron Hassan (Sydney, Australia), Sachin Kalia (Graduate Student, U of Minnesota), Sayantan Khan (Student, India), Rob Kline (W. Lafayette, IN), Joe Klobusicky (Geisinger Health Systems), Peter Kornya (Retired Faculty, Ivy Tech), Tin Lam (Engineer, St. Louis, MO), Steven Landy (Physics Faculty, IUPUI), Wei-Xiang

Lien (Miaoli, Taiwan), Matthew Lim, Philip Nowell (Co-founder, Controlled Panic LLC), Esmaeil Parsa (Lecturer, Iran), Benjamin Phillabaum (Visiting Scholar, Physics, Purdue), Achim Roth (Data Protection Officer, Germany), Sorin Rubinstein (TAU faculty, Tel Aviv, Israel), Shin-ichiro Seki (Graduate Student, Osaka University), Jason L. Smith (Professor, Richland Community College, IL), David Stoner (HS Student, Aiken, S. Carolina), Hakan Summakoglu (Antakya, Turkey), Michael Tomaine (Bellevue, WA), Motohiro Tsuchiya (Graduate student, USUHS), Christopher J. Willy (Part-time Faculty, GWU), William Wu (Quantitative Engineering Design Inc.)