

PROBLEM OF THE WEEK
Solution of Problem No. 3 (Fall 2014 Series)

Problem:

What is the largest possible area of a simple quadrilateral, two sides of which have length a and two sides of which have length b ?

Solution: (by Scott Kelly, Junior, Computer Science, Purdue University)

Pick a side of length a . It shares a vertex with at least one side of length b . Thus, there is a vertex P where one side has length a and the other side has length b . Moreover, the same is true for the opposite vertex, R , since it has the other sides of the quadrilateral. Draw a diagonal between the other vertices, Q and S . Now QPS and QRS are triangles with equal side lengths. The area of each triangle can be found by $A = \frac{1}{2}ab \sin \theta$, where θ is the angle between the sides of length a and b . Maximally, $\sin \theta = 1$ when we have right triangles, so the maximum area of the quadrilateral is $2 \left(\frac{1}{2}ab \right) = ab$.

[Added by the panel: So, if $a \neq b$ there are two different shapes which have the maximum area.]

The problem was also solved by:

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Graduates: Tairan Yuwen (Chemistry)

Others: Vanio Beccaccioli (Terni, Italy), Marco Biagini (Math Teacher, Italy), Charles Burnette (Grad Student, Drexel Univ.), Hongwei Chen (Professor, Christopher Newport Univ. Virginia), Jiehua Chen (Quantitative Engineering Design Inc.), Dylan Cordaro (Don Bosco Prep), Dan-One (Israel), Sandipan Dey (UMBC Alumni), Ghasem Esmati (Sharif Univ. of Tech), Talal AL Fares (Lebanon), Emmanuel A. Garcia (HS Teacher, CA), Rick Shilling & Bruce Grayson (Orlando, FL), Mohammed Hamami (AT & T), Aaron Hassan (Sydney, Australia), Sachin Kalia (Graduate Student, U of Minnesota), Joe Klobusicky (Geisinger Health Systems), Peter Kornya (Retired Faculty, Ivy Tech), Tin Lam (Engineer, St. Louis, MO), Steven Landy (Physics Faculty, IUPUI), Wei-Xiang Lien (Miaoli, Taiwan), Matthew Lim, Vladimir B. Lukianov (Lecturer, Tel-Aviv), Glen Nieman

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