PROBLEM OF THE WEEK
Solution of Problem No. 1 (Spring 2001 Series)

Problem: The shorter leg of an integer-sided right triangle has length 2001. How short can the other leg be?

Solution (by the Panel)

Let a, b, c be the sides of the triangle. Thus $2001 = a < b < c$. Set $c = b + m$. Then $(b+m)^2 = b^2 + 2001^2, m(2b+m) = 2001^2$. So m is a divisor of $2001^2 = 3^2 \cdot 667^2$ and since $b = c - m$ is to be shortest (> 2001), $m = 667$ (the next largest divisor is $3 \cdot 667 = 2001$, which makes $b = 0$) should be considered. Then $667 \cdot (2b + 667) = 9 \cdot 667^2$ gives $b = 2668$ and $c = 2668 + 667 = 3335$. One checks that $2001^2 + 2668^2 = 3335^2$.

Comment: This triangle is the $(3, 4, 5)$ triangle since $(2001, 2668, 3335) = 667(3, 4, 5)$. But recognizing this does not prove that 2668 is the shortest possible side larger than 2001.

Completely or partially solved by:

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Five unacceptable solutions were received.