PROBLEM OF THE WEEK
Solution of Problem No. 3 (Spring 2001 Series)

Problem: Show that $x^4y^2z^2 + x^2y^4z^2 + x^2y^2z^4 - 4x^2y^2z^2 + 1 \geq 0$ for all (x, y, z) in $\mathbb{R}^3$.

Solution (by Wook Kim, Grad. Math)

By the arithmetic and geometric inequality

$$\frac{x_1 + x_2 + \cdots + x_n}{n} \geq \sqrt[n]{x_1 x_2 \cdots x_n}, x_i \geq 0,$$

we have

$$\frac{x^4y^2z^2 + x^2y^4z^2 + x^2y^2z^4 + 1}{4} \geq \sqrt[4]{x^8y^8z^8} = x^2y^2z^2.$$

This proves $x^4y^2z^2 + x^2y^4z^2 + x^2y^2z^4 - 4x^2y^2z^2 + 1 \geq 0$ for all $x, y, z \in \mathbb{R}^3$.

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