## PROBLEM OF THE WEEK

Solution of Problem No. 6 (Spring 2001 Series)

Problem: In the game $G(m, n)$ for given integers $m, n$ with $2 \leq m \leq n$, players A and B alternately subtract any positive integer less than $m$ from a running score which starts at $n$. Player A starts, and the winner is the player who brings the score to zero. For given $m, n$ there is always one player who can force a win. Find who, and explain how.

Solution (by Steven Landy, Fac. Physics at IUPUI; this solution is essentially the same as that of the other solvers)

A winning position is to leave your opponent with a multiple of $m$, who must then leave you with an amount not equal to a multiple of $m$. Continuing in this way you eventually leave 0 (a multiple of $m$ ). Thus if $n \not \equiv 0(\bmod m)$, A can force a win, while if $n \equiv 0(\bmod m), \mathrm{B}$ can force a win.

Remark. Some solvers interpreted the problem to say that $m \leq n$ rather than $m<n$. In that case the solution depends on whether $n \equiv 0$ or $\not \equiv 0(\bmod m+1)$.

Also solved by:
Undergraduates: Eric Tkaczyk (Jr. MA), Yee-Ching Yeow (Jr. Math)
Graduates: Ashish Rao (ECE), Dharmashankar Subramanian (CE)
Others: Jake Foster (Soph. Harrison H.S., WL), Jonathan Landy (Jr. Warren Central H.S., Indpls.), Mr. Rice's Class (E. Tipp Middle Sch., Laf), Julien Santini (Lacordaire H.S., France)

One unacceptable solution was received.

