## PROBLEM OF THE WEEK

Solution of Problem No. 7 (Spring 2001 Series)

Problem: Let $C$ be a smooth closed curve (no corners) in the plane with a convex interior, and $P$ a given point on $C$. Show that there are points $Q, R$ on $C$ such that $\triangle P Q R$ is equilateral.

Solution (by Julien Santini, Lacordaire H.S., France; edited by the Panel)
Let an angle of $60^{\circ}$ revolve counter-clockwise about $P$, with initial position of one of the arms tangent to $C$ at $P$. The intercepts of the two arms are initially 0 and some $q>0$. Turn the angle until the other arm becomes tangent to $C$, and the intercepts are now some $r>0$ and 0 . Hence the difference of the intercepts changes from $0-q<0$ to $r-0>0$. By continuity there is a position of the two arms $\overline{P Q}, \overline{P R}$ where $|P Q|=|P R|$, hence $\triangle P Q R$ is equilateral.

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