## PROBLEM OF THE WEEK Solution of Problem No. 7 (Spring 2001 Series)

**Problem:** Let C be a smooth closed curve (no corners) in the plane with a convex interior, and P a given point on C. Show that there are points Q, R on C such that  $\triangle PQR$  is equilateral.

Solution (by Julien Santini, Lacordaire H.S., France; edited by the Panel)

Let an angle of 60° revolve counter-clockwise about P, with initial position of one of the arms tangent to C at P. The intercepts of the two arms are initially 0 and some q > 0. Turn the angle until the other arm becomes tangent to C, and the intercepts are now some r > 0 and 0. Hence the difference of the intercepts changes from 0 - q < 0 to r - 0 > 0. By continuity there is a position of the two arms  $\overline{PQ}$ ,  $\overline{PR}$  where |PQ| = |PR|, hence  $\triangle PQR$  is equilateral.

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