## PROBLEM OF THE WEEK Solution of Problem No. 10 (Spring 2001 Series)

**Problem:** Determine the positive numbers a such that  $\sqrt[3]{3 + \sqrt{a}} + \sqrt[3]{3 - \sqrt{a}}$  is an integer.

Solution (by Yee-Ching Yeow, Jr. MA)

Let 
$$n = \sqrt[3]{3 + \sqrt{x}} + \sqrt[3]{3 - \sqrt{x}}$$
 for  $x > 0$ . Then  $n^3 = 6 + 3[(3 + \sqrt{x})(3 - \sqrt{x})]^{1/3} n$ , hence  
 $\left(\frac{n^3 - 6}{3n}\right)^3 = 9 - x, \quad x = 9 - \left(\frac{n^3 - 6}{3n}\right)^3 > 0.$ 

Since  $\left(\frac{n^2}{3} - \frac{2}{n}\right)^3$  is monotone increasing and larger than 9 for  $n \ge 3$ , it suffices to let n be 1 or 2. When n = 1, x = 368/27, and when n = 2, x = 242/27.

Also solved by:

Undergraduates: Stevie Schraudner (Jr. CS/MA), Eric Tkaczyk (Jr. EE/MA)

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<u>Others</u>: Damir D. Dzhafarov (Sr. Harrison H.S., WL), Julien Santini (Lacordaire H.S., France)

Three incorrect solutions were received.