## PROBLEM OF THE WEEK

Solution of Problem No. 10 (Spring 2001 Series)

Problem: Determine the positive numbers $a$ such that $\sqrt[3]{3+\sqrt{a}}+\sqrt[3]{3-\sqrt{a}}$ is an integer.

Solution (by Yee-Ching Yeow, Jr. MA)
Let $n=\sqrt[3]{3+\sqrt{x}}+\sqrt[3]{3-\sqrt{x}}$ for $x>0$. Then $n^{3}=6+3[(3+\sqrt{x})(3-\sqrt{x})]^{1 / 3} n$, hence

$$
\left(\frac{n^{3}-6}{3 n}\right)^{3}=9-x, \quad x=9-\left(\frac{n^{3}-6}{3 n}\right)^{3}>0
$$

Since $\left(\frac{n^{2}}{3}-\frac{2}{n}\right)^{3}$ is monotone increasing and larger than 9 for $n \geq 3$, it suffices to let $n$ be 1 or 2 . When $n=1, x=368 / 27$, and when $n=2, x=242 / 27$.

Also solved by:
Undergraduates: Stevie Schraudner (Jr. CS/MA), Eric Tkaczyk (Jr. EE/MA)
Graduates: Sridhar Kompella (IE), Chris Lomont (MA), Ralph Shines (GAANN Fellow, MA)

Faculty \& Staff: Steven Landy (Phys. at IUPUI), William Wolber Jr. (PUCC)
Others: Damir D. Dzhafarov (Sr. Harrison H.S., WL), Julien Santini (Lacordaire H.S., France)

Three incorrect solutions were received.

