PROBLEM OF THE WEEK
Solution of Problem No. 1 (Spring 2002 Series)

Problem: Given an integer N whose decimal representation consists of 2001 2's preceeded and followed by a 1. Determine the highest power of 11 that divides N.

Solution (by the Panel)

Use the divisibility by 11 criterion (well known and easily proved): The decimal number N is divisible by 11 if and only if the sum of the 1st, 3rd, 5th, ... digits minus the sum of the 2nd, 4th, 6th, ... digits is divisible by 11.

The criterion shows that the given N is divisible by 11. A simple division yields $122\ldots21 \div 11 = 111\cdots1$, a number of 2002 ones. Again by the criterion, this number is divisible by 11. Now $11\ldots11 \div 11 = 1010\cdots101$, a number with 1001 ones and 1000 zeros. 1001 is divisible by 11, hence N divisible by 11^3. Now $1010\ldots01 = 10^{2000}+10^{1998}+\cdots+10^2+1 \equiv 1+1+\cdots+1 = 1001 \pmod{11}$. Also $1001 \div 11 = 91$, which is not divisible by 11. So 11^3 is the largest power of 11 that divides N.

Solved by:

Undergraduates: Stevie Schraudner (Sr. CS/MA)

Graduates: Chris Lomont (MA)

Faculty: Steven Landy (Phys. at IUPUI)

Others: Jake Foster (Jr. Harrison H.S., WL), Jonathan Landy (Warren Central H.S., Indpls), Peter Montgomery (San Rafael, CA), Unidentified (paper without a name)

Four unacceptable solutions were received.