PROBLEM OF THE WEEK

Solution of Problem No. 7 (Spring 2002 Series)

Problem: There are 9 points in the interior of a cube of side 1.

a) Show that at least two of them are less than $\frac{1}{2}\sqrt{3}$ apart.

b) Can $\frac{1}{2}\sqrt{3}$ be replaced by a smaller number?

Solution (by Damir Dzhafarov, Fr. MA)

a) The cube may be partitioned into eight equal cubes, each with side of $1/2$, so that at least one of these cubes contains more than one point. The greatest distance between any two points in the interior of a cube is less than the length of its longest diagonal, which in the case of the smaller cubes is precisely $\frac{1}{2}\sqrt{3}$.

b) Placing a point at the center of the unit cube, and the remaining ones arbitrarily close to the cube's vertices yields, among the distances between any two points, one arbitrarily close to $\frac{1}{2}\sqrt{3}$. Hence, this number cannot be replaced by a smaller one in the result of (a).

Also solved by:

Undergraduates: Haizhi Lin (Jr. MA), Eric Tkaczyk (Jr. EE/MA)

Graduates: Gajath Gunatillake (MA)

Faculty: Steven Landy (Phys. at IUPUI)

Others: Rob Pratt (Gr. U.N.C., Chapel Hill, NC), Henry Shin (Sr. Fairfax H.S., LA, CA), Minseou Shin (5th gr. 3rd St. School, LA, CA), Aditya Utturwar (Grad. AE, Georgia Tech)

One correct but unidentified solution of problem 7 was received. Three unsatisfactory solutions were received.

A correct solution of problem 6 by Peter Montgomery from CA was probably on time.

We have received 8 late solutions to problem 5 which we will report on later.