

PROBLEM OF THE WEEK
Solution of Problem No. 1 (Spring 2003 Series)

Problem: Show that the binary number $t = .111 \cdots 1$ with 2003 1's satisfies $.99 \cdots 9 < t < .99 \cdots 9$ where the lower bound has 602 decimal digits 9 and the upper bound has 603 decimal digits 9.

Solution (by Jason Andersson, Soph. Math.)

$$\begin{aligned} 0.301 &< \log_{10} 2 < 0.30103, \quad \text{hence} \\ 602 &< 2003 \log_{10} 2 < 603, \quad \text{thus} \\ 10^{602} &< 2^{2003} < 10^{603} \quad \text{and} \\ 1 - 10^{-602} &< 1 - 2^{-2003} < 1 - 10^{-603}, \quad \text{or} \\ \underbrace{0.999 \cdots 9}_{602} &< \underbrace{(0.111 \cdots 1)_2}_{2003} < \underbrace{0.999 \cdots 9}_{603} \end{aligned}$$

Also solved by:

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