PROBLEM OF THE WEEK

Solution of Problem No. 2 (Spring 2003 Series)

Problem: Let T be a triangle in 3-dimensional Euclidean space. Show that the sum of the squares of the areas of the three triangles which are the projections of T onto three mutually orthogonal planes is independent of the location of the planes.

Solution (by Steven Landy, Fac. Physics at IUPUI)

We may represent the area of a triangle T as a vector $\vec{A}$ in the xyz-system whose length is the value $|A|$ of the area and whose direction is orthogonal to the plane of the triangle. The projection of T onto a plane is a triangle whose area is $|\vec{A}\cdot\vec{n}|$, where $\vec{n}$ is either unit vector normal to the plane. Hence the projected areas are the absolute values of the components A_x, A_y and A_z of $\vec{A}$, and since these are orthogonal we have

$$A_x^2 + A_y^2 + A_z^2 = |A|^2,$$

hence independent of the position of the xyz-system.

Comment (by the Panel). The result is a true extension of the Pythogorean Theorem: Given a segment S (a 1-dimensional simplex), the square of the length (area in 2 dimensions) is the sum of the squares of the lengths of the two segments which are the projections of S onto two orthogonal lines.

Also solved by:

Undergraduates: Jason Andersson (Fr. MA), Ryan Machtmes (Sr. E&AS), Neel Mehta (Fr. AAE)

Graduates: Yifan Liang (ECE), Ashish Rao (ECE),

Others: J.L.C. (Fishers, IN), Vijay Madhavapeddi (Newark, CA)

One unacceptable solution was received.