PROBLEM OF THE WEEK

Solution of Problem No. 3 (Spring 2003 Series)

Problem: Given $b_1 = 1$, $b_n = 1 - \frac{1}{n^2}\sum_{k=1}^{n-1} k^2 b_k$ for $n \geq 2$, sum the series $\sum_{n=1}^{\infty}(-1)^{n-1}b_n$. You may use that $\sum_{n=1}^{\infty}(-1)^{n-1}/n^2 = \pi^2/12$.

Solution (by Namig Mammadov, Baku, Azerbaijan)

$$\sum_{k=1}^{n} k^2 b_k = \sum_{k=1}^{n-1} k^2 b_k + n^2 b_n = n^2 - n^2 b_n + n^2 b_n = n^2,$$

so we get

$$b_n = 1 - \frac{1}{n^2}\cdot(n-1)^2 = \frac{2n-1}{n^2} = \frac{2}{n} - \frac{1}{n^2}.$$

Hence

$$\sum_{n=1}^{\infty}(-1)^{n-1}b_n = \sum_{n=1}^{\infty}(-1)^{n-1}\left(\frac{2}{n} - \frac{1}{n^2}\right) =$$

$$2\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n} - \sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^2} = 2\ln 2 - \frac{\pi^2}{12}.$$

Also solved by:

Undergraduates: Jason Andersson (Fr. MA), Ryan Machtmes (Sr. E&AS), Neel Mehta (Fr. AAE) Yen Hock Tan (Fr. CS)

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There was one unacceptable solution.