PROBLEM OF THE WEEK

Solution of Problem No. 7 (Spring 2003 Series)

Problem: Prove that every planar compact set of at least two points has a circumscribed square. (A set is compact if it is bounded and contains all its limit points. Set S is circumscribed by square Q if $S \subseteq Q$ and every side of Q contains at least one point of S.)

Solution (by Steven Landy, staff Physics at IUPUI)

For every angle θ from a fixed reference line ℓ there are two supporting lines of Q making angle θ with ℓ, and two other supporting lines making angle $\theta + \frac{\pi}{2}$ with ℓ. Let $f(\theta)$ denote the distance between the supporting lines for angle θ. Define $g(\theta) = f(\theta) - f(\theta + \frac{\pi}{2})$; then g is a continuous function and it changes from a value d at θ to $-d$ at $\theta + \frac{\pi}{2}$. By the Intermediate Value Theorem $g = 0$ at some θ_x between θ and $\theta + \frac{\pi}{2}$. The supporting lines at θ_x and $\theta_x + \frac{\pi}{2}$ from a circumscribed square of Q.

Also solved by:

Graduates: Thierry Zell (MA)

Others: Regis J. Serinko (PhD, State Coll., PA)

One incorrect solution was received.